Assignment #1 part 1, Total points: 50
(Course: CS 401)

These are the first two problems for Assignment 1. **The remaining problems of Assignment 1 will be given out later.**

For regular students, the deadline is **February 14, Wednesday, in class**.

For special needs students, the deadline is **February 21, Wednesday, in class**.

No late assignments will be accepted.

Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 1 (20 points): Consider the stable matching problem as taught in class. Suppose that we have only three men, say m_1, m_2 and m_3, and only three women, say w_1, w_2 and w_3, with their corresponding preference lists. Suppose also that the matching m_1–w_1, m_2–w_2, m_3–w_3 is a stable matching. We make the following claim:

> in this case the matching m_1–w_1, m_2–w_3, m_3–w_2 can **never** be a stable matching.

Your task is to decide if our claim is true or false. For this purpose, do the following.

- Either prove the claim is indeed correct. Such a proof should work **no matter** what the preferences of the men and women are, as long as m_1–w_1, m_2–w_2, m_3–w_3 is a stable matching.
- Or, prove the claim made is wrong by giving a counter-example. The counter-example should provide the preferences lists of every man and woman, and show that for these preference lists **both** m_1–w_1, m_2–w_2, m_3–w_3 and m_1–w_1, m_2–w_3, m_3–w_2 are indeed stable matchings.

Problem 2 (30 points): Assume that you have two functions $f(n)$ and $g(n)$ such that $f(n) = O(g(n))$. Also, assume that $f(n) \geq 1$ and $\log_2 g(n) \geq 1$ for all n. For each of the following statements, decide whether you think it is true or false and accordingly give a proof (if true) or a counter-example (if false).

***(i)* (10 points)** $\log_2 f(n)$ is $O(\log_2 g(n))$.

***(ii)* (10 points)** $2^{f(n)}$ is $O\left(2^{g(n)}\right)$.

***(iii)* (10 points)** $f(n)^3$ is $O\left(g(n)^3\right)$.