Measuring the Efficiency of Urban Taxi Service System

Xianyuan Zhan
Purdue University
550 Stadium Mall Dr
West Lafayette, IN
zhanxianyuan@purdue.edu

Xinwu Qian
Purdue University
550 Stadium Mall Dr
West Lafayette, IN
qian39@purdue.edu

Satish V. Ukkusuri
Purdue University
550 Stadium Mall Dr
West Lafayette, IN
sukkusuri@purdue.edu

ABSTRACT
The taxi service systems in big cities are immensely complex due to the interaction and self-organization between taxi drivers and passengers. An inefficient taxi service system leads to more empty trips for drivers and longer waiting time for passengers, and introduces unnecessary congestion to road network. Although understanding the performance of urban taxi service system is important, the performance of the urban taxi service is rarely examined. In this experimental paper, we investigate the efficiency level of the taxi service system using real world large-scale taxi trip data. By assuming a hypothetical system-wide recommendation system, two approaches are proposed to find the theoretical optimal strategy that minimizes the cost of vacant trips, and results in minimum number of taxis required to satisfy all observed trips. The optimization problems are transformed into equivalent graph problems and solved using polynomial time algorithms. The taxi trip data in New York City is used to quantitatively examine the gap between the current system performance and the theoretically optimal system. The numerical results indicate that if a centralized system-wide recommendation scheme applied, it is possible to reduce 20% to 90% total vacant trip cost depending on different objectives, and 1/3 of all taxis that required to serve all observed trips. The huge performance gap obtained suggests an urgent need for the system reconsideration in taxi recommendation system design.

Keywords
Taxi service system, large-scale trip data, efficiency, graph theory

1. INTRODUCTION
The urban taxi service is a special component of public transportation system. The door-to-door service and the 7-24 availability complete the gaps of the other public transport modes. In New York City (NYC), the 13,000 yellow medallion taxis serves 240 million passengers per year and transports 71% of all Manhattan residents' trips [1]. In Hong Kong, there are 15,000 taxicabs by the end of 2013 and more than 1,000,000 passengers are served everyday [2]. While up to 60% of the daily flow in particular areas in Hong Kong are generated by taxis, however, most of them are empty trips [3]. The excessive vacant trips increase the waiting time for passengers and the operating cost for drivers. Meanwhile, it also introduces negative externalities such as unnecessary congestion and emission [4]. Hence, the taxi market should be optimized to make more efficient use of the available resources.

Precursory approaches tend to address the efficiency issue of the taxi service system at the aggregate level (total demand and supply), where economic relationships are considered to determine the optimal fare setting and fleet size [5, 6, 7]. More realistic models are further proposed to account for taxi cruising behavior, demand elasticity and network congestions [8, 9, 10, 11]. All these studies provide a preliminary direction to enhance the system performance by introducing the supply regulation to the taxi market. The limitation, however, comes from the unrealistic assumption of the taxi driver and passenger's behavior and the inability to fully characterize the taxi service system using overly abstracted mathematical models. The taxi service system is an immensely complex self-organized system: drivers are self-adaptive based on their own knowledge about the traffic, and the passenger demand is both spatially and temporally varying. For example, drivers are likely to roam near residential areas during morning peak and wait at the concert when approaching the end of a play. The inefficiency of the system arises, even when the market is properly regulated, due to the lack of perfect system-wide information sharing between taxi drivers and passengers.

In the era of big data, the development of the sensing technologies and large-scale computing infrastructures give birth to the idea of the taxi dispatching, recommendation, and ridesharing service systems [12, 13, 14, 15, 16, 17, 18]. By clustering historical pick-up locations based on the temporal and spatial characteristics, suggestions for drivers are provided to reduce the empty trips [13]. The cruising distance before finding a passenger can be reduced by learning from experienced drivers and making a sequence of recommendations [14]. Yamamoto et al. [15] proposed a fuzzy clustering and adaptive routing algorithm to dispatch vacant taxis to places where passengers are more likely to be found. In addition to aid taxi drivers, Yuan et al. [16] also incor-

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porated the passengers' mobility patterns and taxi driver's picking-up/dropping-off behaviors to provide recommendations for passengers to reduce disequilibrium between the demand and supply. Apart from improving the taxi services through dispatching passenger and vacant taxis, or providing guidance to taxi drivers, ridesharing services is another way to reduce congestion and energy consumption. Several works on ridesharing systems [17, 18] have been developed with the consideration of time, capacity and monetary constraints. Despite the technological advances in ridesharing services, non-technical problems remain, such as the credit of a passenger and taxi driver as well as security issues [12].

The geo-location data is seen as the remedy for the asymmetric information between drivers and passengers and is shown to be helpful for more efficiently taxi and passenger matching. While some drivers or passenger are benefited from the aforementioned taxi dispatching and recommendation scheme, whether the efficiency of the entire system is improved remains questionable. Prerequisite and fundamental research questions to be answered are how efficient is the current taxi service system, and how to quantitatively evaluate the performance level of the current system. Moreover, if a theoretically optimal system exists, how far is the performance gap between the current system and the theoretical optimum?

In this paper, we propose two different approaches to find the theoretical optimum for the taxi utilization at the system level. The corresponding optimization problems are transformed into equivalent graph problems and solved using polynomial time algorithms in graph theory. The first approach targets at providing the optimal matching strategy between vacant taxis and possible passengers, which is transformed and solved as a minimum weight perfect bipartite matching problem. The second approach aims to find the optimal integration strategy of sequences of trips for the taxis in the system, which is showed to be equivalent to a minimum path cover and minimum weighted minimum path cover problem for unweighted and weighted sub-problems respectively. Due to previously discussed non-technical issues for ridesharing services, this work focuses on analyzing ordinary taxi trips, and taxi ridesharing is not considered. To the best of our knowledge, it is the first work to quantitatively examine the efficiency of the entire taxi network using large-scale real world taxi data.

The rest of the paper is structured as follows. The next section describes the data used for this paper; section 3 presents the methodological approach developed to find the theoretically optimal strategy; section 4 shows the experiment results and the the concluding remarks are given in the final section.

2. DATA

The data used in this research were collected by the New York City Taxi and Limousine Commission (NYCTLC) on the trip by trip basis. The data contains the taxi medallion ID, driver initial, shift number, the timestamp and geographical location for trip origin and destination, trip duration, travel distance and fare etc. Around 30,000 to 50,000 daily trips were recorded during the data collection period from December, 2008 to January 2010. Observing the stable trip pattern during weekdays and weekend [19], one-week data from October 5th, 2009 to October 11th, 2009 is extracted for further analysis, in which no major social events were reported. We focus on investigating the efficiency of the taxi service system on a typical weekday (2009-10-07, Wednesday) and day of the weekend (2009-10-10, Saturday).

There are more than 13,000 medallion cabs in NYC [1], but only 818 unique medallion IDs are found in the data, which is far from the right number. As a result, the first step is to recover the corresponding trip set for each of taxi. By observation, a specific medallion ID is associated with multiple driver initials and shift numbers. Therefore, the combination of taxi medallion ID, driver initial and shift number can be used as a unique taxi identifier (referred as taxi ID). All trips under a same taxi ID are retrieved and sorted based on trip starting time. The vacant trip information can thus be obtained by comparing the consecutive drop-off and pick-up locations and timestamps. For each taxi, since it is hardly possible to gain information prior to its first trip, only the drop-off location of the first trip is utilized and served as the initial position of the taxi.

For the weekday data, we observe a total of 489,234 trips within a day, and 32,368 (6.6%) distinct taxis are recovered. For the weekend data, 33,999 taxis are recognized from the 524,792 trips. Considering that there are usually two or three daily shifts take place for each taxi, the number of taxis recovered is very close to the actually number of medallion cabs in NYC. .

3. METHODOLOGY

This section presents the details of the two approaches used to evaluate the efficiency of the taxi service system, namely the optimal matching and the trip integration. Consider following two aspects of the efficiency in the taxi service system:

• If the information for both the taxis and passengers are known within a time interval, how to match the given set of vacant taxis and the passengers so that the total time/distance or revenue loss are minimized?

• If a given set of taxi trips are known beforehand, how to combine a sequence of trips served by individual taxis to achieve the minimum utilization of the number of taxis and lower the total trip cost?

The first research question associates with the issue that taxi drivers tend to spend too much time or travel extra miles than actually needed to find the next passenger. The second research question is particularly meaningful if there is a high demand for taxis while the supply is very limited, such as during peak hours. Apparently, the system efficiency will be improved by addressing either of the two problems. The optimal matching provides an optimal solution for the first scenario while the second one is addressed by the trip integration.

3.1 Assumptions

There are four major assumptions related to the methodology:

1. There exists a hypothetical system-wide recommendation mechanism for performing the optimal matching and trip integration for every time interval T with a length of ΔT, within which the information of both taxis and passenger trips are known. The efficient matching will require a short ΔT while a longer ΔT is expected to have enough trips for the integrationčż
2. The starting trips are only considered as a starting point and are not involved in the matching cost computationčż
3. All trips observations must be served at the exact time and location as recorded in the datačż
4. The computation of travel distance between consecutive trips (vacant trip distance) is simplified as the Euclidean distance, due to the missing trajectory information and the huge amount of data.

3.2 Notation

The notations used in the mathematical formulations are given as follows:

T	Index of the current time interval, $T \in \{1, 2, 3..., \mathbb{T}\}$;
ΔT	Length of the time interval;
A^T	The set of all available taxis at time interval T. All taxis finish serving a trip within T are seen as available, except the observed last trip of a taxi. Each available taxi in A^T is represented as a tuple: (i, p_i^a, t_i^a), corresponding to the taxi ID, location and timestamp when available (end point and timestamp of last trip)čż
B^T	The set of trips to be served in interval T. Each trip in B^T is represented as a tuple: $(j, p_j^o, t_j^o, p_j^d, t_j^d)$, corresponding to the trip ID, location and timestamp of trip origin, location and timestamp of trip destination respectively;
R^T	The set of unserved taxi in time interval T. The cardinality of R^T is $\lvert R^T\rvert = \lvert A^T\rvert - \lvert B^T\rvert$;
M	A sufficient large number;
z_{ij}^T	Possible matching between taxi $i \in A^T$ and trip $j \in B^T$;
d_{ij}^T	The Euclidean distance of the matching between p_i^a and p_j^o, $d_{ij}^T = \lvert\lvert p_i^a - p_j^a \rvert\rvert_2$;
α, β	Cost coefficients for the distance and time of vacant trips;
w_{ij}^T	Cost for matching taxi $i \in A^T$ and trip $j \in B^T$;
$G(V,E)$	Graph notation with the set of vertices $V = \{v_k\}$ and the set of edges $E = \{e_{ij}\}$;
c_{ij}^T	Capacity of edge $e_{ij} \in E$;
v_{max}	Maximum possible travel speed for matching. Taken as 20 mile/hour in actual implementation.

3.3 Optimal Matching

Given the time interval ΔT (assumed to be small enough so that no more than one complete trip is finished during the interval), the objective of the optimal matching is to find the optimal strategy so that the total matching cost is minimized. The matching cost can be measured as the taxi idle time, vacant trip distance or revenue loss which is the weighted combination of the previous two. The optimal matching problem for each interval $T = 1, 2, ..., \mathbb{T}$ can be formulated as the following integer linear program (ILP):

$$Min \sum_{ij} w_{ij}^T z_{ij}^T \tag{1}$$

$$s.t. \sum_i z_{ij}^T = 1, \quad i \in A^T \tag{2}$$

$$\sum_j z_{ij}^T = 1, \quad j \in B^T \bigcup R^T \tag{3}$$

$$w_{ij}^T = \begin{cases} \alpha(t_j^o - t_i^a) + \beta d_{ij}^T, & if j \in B^T \ ; \\ M, & \text{o.w.} \end{cases} \tag{4}$$

$$z_{ij}^T \in \{0, 1\} \tag{5}$$

Equation (2) and (3) ensure the one to one mapping (perfect matching) between taxis and trips. Equation (4) defines the matching cost. Equation (5) restricts z_{ij}^T to be a binary variable. $z_{ij}^T = 1$ if a matching exist, and 0 otherwise. The z_{ij}^T and w_{ij}^T exist if and only if the following conditions are satisfied:

$$t_j^o \geq t_i^a \tag{6}$$

$$d_{ij}^T = (t_j^o - t_i^a) v_{max} \tag{7}$$

Equation (6) states that the taxi available time should be no later than the trip starting time. Equation (7) sets the maximum Euclidean distance between available taxi location and trip starting location for the valid matching.

If we abstract the set of taxis and the set of trips as two sets of vertices, and the set of valid matching as the set of edges, the ILP problem can be represented as a bipartite graph as illustrated in Figure 1. Moreover, there is a cost w_{ij}^T associated with each pair of matched taxi and passenger trip, which is considered as the weight of the corresponding edge. We can show the ILP is equivalent to the minimum weight perfect bipartite matching problem.

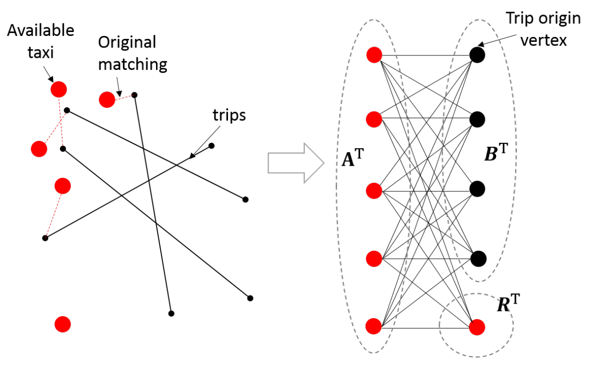


Figure 1: Illustration of Optimal Matching

Definition 1. Given a bipartite graph $G = (V, E)$ with bipartition S and T ($V = S \bigcup T$) and weights w_{ij} for all edges $e_{ij} \in E$ ($E = S \times T$), a **bipartite matching** is a subset of edges $M \subseteq E$, such that for all vertices $v \in S \bigcup T$, at most one edge of M is incident on v. A matching is **perfect** if no vertex is exposed. The **minimum weight perfect bipartite matching** is to find a perfect matching of minimum cost. This problem is also called the *assignment problem.*

From above definition, it can be easily observed that the ILP defined by Equation (1)-(5) is exactly equivalent to the minimum weight perfect bipartite matching problem. Since in this experiment, we dedicate to find the optimal matching from the observed taxi trips, thus at least one matching (the actual one) exists for the problem. Hence the given problem is always solvable, and there is at least one available taxi exists for each trip (indicates $|A| \geq |B|$, and $|R^T| = |A^T| - |B^T|$), thus the perfect matching is always well defined. Note that when applied to real-world cases, the perfect matching may not always exists, as $|A| < |B|$ may occur. However the problem can still be solved as a minimum weight bipartite matching problem. The celebrated Hungarian method [20] is known to solve the minimum weight bipartite matching problem, which has a complexity of $O(|V|^3)$. For detailed description about minimum weight bipartite matching and the Hungarian method, please refer to [21].

By solving the optimal matching problem and obtain the optimal solution $z_{ij}^{T^*}$,the optimal matching strategy can be retrieved and the set of available taxis A^T is updated for the next interval $T+1$ as follows:

1. If $j \in B^T$, then taxi i is matched to trip j, and trip j is not the last trip of a taxi. Set $t_{i'}^a = t_j^d$ and $p_{i'}^a = p_j^d$, where $i' \in A^{T+1}$. The optimized vacant trip idle time and distance are computed as $t_k^v = t_j^o - t_i^a$ and $d_k^v = d_{ij}$;
2. If $i \in R^T$, then taxi i is not matched to any trip and is kept for next time interval. Set $t_{i'}^a = t_i^a$ and $p_{i'}^a = p_i^a$,where $i' \in A^{T+1}$.

For each trip $k = 1, 2, 3...n^T$ and each time interval T, the total matching cost is calculated as:

$$\sum_{T=1}^{\mathbb{T}} \sum_{k=1}^{n^T} \alpha t_k^v + \beta d_k^v \tag{8}$$

Specifically, if:

1. $\alpha = 1, \beta = 0$: the objective is to minimize the total taxi idle time;
2. $\alpha = 0, \beta = 1$: the objective is to minimize the total vacant trip distance;
3. $\alpha = \alpha_0, \beta = \beta_0$: the objective is to minimize the total revenue loss from vacant trips, where α_0, β_0 are the cost coefficient of time and distance components.

We build a simple linear regression model for taxi fare with the dependent variable of the travel time and distance using 415,561 taxi trip records. The linear model fits quite well with the data, with highly significant parameters and a $R^2 = 0.99$. The model is presented as follows and the results are presented in Table 1:

$$fare = \alpha_0 \cdot time + \beta_0 \cdot distance \tag{9}$$

3.4 Trip Integration

Given a time interval ΔT and a set of observed trips, the objective of the trip integration is to find an optimal trip combination (integration) strategy that: (1) results in the minimum number of taxis required to satisfy all the trips; and (2) results in minimum total matching cost while achieving minimum possible number of taxis satisfying all the trips. The notion of the trip integration is different from the usual trip merging or combination in ridesharing problems [18], since the all trips combined in this problem are complete trips. The trip integration can be especially beneficial in cases such as peak hours, when a limited number of taxis need to serve more potential passengers. By introducing the trip integration, the resources for taxis will be fully utilized and the system output can be maximized.

Table 1: Linear Regression Model for Taxi Fare-time-distance Relationship

	Coefficient	Standard Deviation	P-value
constant	2.143	0.0016	0.000
α_0 (min)	0.275	0.0002	0.000
β_0 (mile)	1.563	0.0006	0.000
Number of observations		415561	
R^2		0.99	

In the trip integration, we need a longer ΔT (e.g. 10min) compared with the optimal matching. Since a too short ΔT will result in too few trips for integration, which may not lead to any significant improvement to the system. We assume for each time interval, all passengers provides their trip information and the trip travel time is known or can be estimated at the beginning of the time period. Two rules are introduced to verify if two trips $(i, p_i^o, t_i^o, p_i^d, t_i^d)$ and $(j, p_j^o, t_j^o, p_j^d, t_j^d)$ are possible to be combined (integrated), we call such trips *combinable trips* if:

1. $0 \leq t_j^o - t_i^d \leq \Delta D$, where ΔD is the maximum delay allowed;
2. $d_{ij}^T \leq (t_j^o - t_i^d) v_{max}$, that the taxi is possible to reach the passenger given the observed distance and time.

In order to formulate and solve the described optimization problem, we transform the original problems into corresponding graph problem. Particularly, we will consider two cases: (1) unweighted trip integration, which finds the minimum number of taxis required to satisfy all the trips; and (2) weighted trip integration, which finds the minimum total matching cost while achieving minimum possible cardinality set of taxis to satisfy all the trips.

3.4.1 Unweighted trip integration

We first transform the unweighted version of the problem into a graph representation. Denote abstracted vertex $v_i^o = (p_i^o, t_i^o)$ and $v_i^d = (p_i^d, t_i^d)$ for the origin and destination location and time tuple of trip i. For each trip, there is an edge connects v_i^o and v_i^d. Furthermore, we add an directed edge between v_i^d and v_j^o if trip i and trip j are combinable (represented as dash line in $G(V, E)$ of Figure 2(a)). Hence the original unweighted trip integration problem is to find a set of connecting edges between trips that form a set of paths that covering all the trips with the minimum cardinality. If we abstract each trip as a single vertex, let $v_i = (v_i^o, v_i^d)$, and only consider the directed edges that connects combinable trips, then we obtain the directed graph $G'(V', E')$ shown in Figure 2(b). It can be shown that the unweighted trip integration problem is equivalent to find a *minimum path cover* on G'.

Definition 2. Given a directed graph $G = (V, E)$, the **minimum path cover** is to find the minimum number of

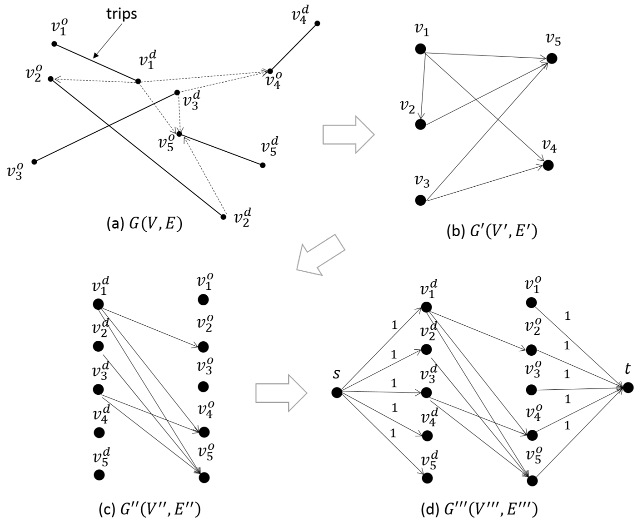


Figure 2: Illustration of Unweighted Trip Integration

paths such that every vertex $v \in V$ belongs to exactly one path. Zero length path is allowed, which is a single vertex.

The reason for the equivalency is straightforward. Since a path is formed only when every consecutive vertices belongs to it are combinable trips. The minimum path cover finds a set of paths that ensure every vertex belongs to a disjoint path, thus all the trips are served, and each trip are served by exactly one taxi. The cardinality is the minimal, thus we find the minimum number of taxis that serve all the required trips, and the paths found by the minimum path cover will be the optimal integrated trips.

Although the minimum path cover problem in general is NP-hard (a path cover has cardinality 1 if and only if the directed graph has a Hamiltonian path), it is solvable in polynomial time on directed and acyclic graphs. For our problem, since the directed edges connects combinable trip vertices, and by the definition of combinable trips, along any existing path, the trip origin timestamp t_i^o will always be increasing. Consequently, the equivalent directed graph G' will never become cyclic.

To solve the minimum path cover problem defined on graph G', we create an equivalent bipartite graph $G''(V'',E'')$ as shown in Figure 2(c). To perform the transformation, we partition all vertices v_i^d and v_i^o into two sets. The edges that connects the combinable trips between v_i^d and v_j^o are also included. It can be shown that by solving a maximum bipartite matching on G'', we solve the minimum path cover problem. To see this, we introduce following definition and proposition:

Definition 3. A **maximum matching** is a matching M on $G(V,E)$ such that every other matching M' satisfies $|M'| \leq |M|$.

PROPOSITION 1. *The directed bipartite graph $G''(V'',E'')$ has a matching of size $n-k$ ($n = |V'|$) if and only if there are k directed paths covering all the vertices in G'.*

PROOF. Assuming the directed acyclic graph $G'(V',E')$ has a path cover P of size k, and let $P_1, P_2, \ldots, P_k$ be the k paths. Hence in the transformed bipartite graph G'', for each path P_i, there will be $|P_i|-1$ matching edges used. Then there is a matching of size $\sum_i^k(|P_i|-1) = \sum_i^k |P_i| - k = n-k$. On the other hand, if G'' has a matching of size $n-k$, it will form m disjoint paths $P_1, P_2, \ldots, P_m$ and l corresponding isolated vertices (both v_i^o and v_d^i are not connected to any other vertices) in G'. Hence these disjoint paths and isolated vertices will form a path cover P on G'. Since $\sum_i^m |P_m| + l = n$ and $\sum_i^m(|P_i|-1) = n-k$, thus $m+l = k$, and P is a path cover of size k. $\square$

A well-known solution approach for maximum bipartite matching is to use the max-flow algorithm with a simple graph transformation [22], which is shown in Figure 2(d). We add dummy source and sink node s, t that connects to all v_i^d and v_i^o respectively, and set capacity of all edges to be 1. Solving the max-flow problem on G''' between s and t will find the maximum bipartite matching on G''. According to Proposition 1, finding the maximum matching on G'' will lead to the minimum size of path cover on G' (maximize $n-k$ is equivalent to minimize k). Hence the unweighted trip integration problem can be efficiently solved using polynomial time max-flow algorithm, specifically, if the Edmonds and Karp algorithm is used, the computation complexity is $O(|E''|^2)$.

3.4.2 *Weighted trip integration*

If we consider each edge $e'_{ij} \in E'$ in Figure 2(b) to be associated with the weight w_{ij}^T, then the problem of finding the minimum number of taxis can be extended to the problem of finding the minimum total matching cost using the least number of taxis. The equivalent graph of this problem is similar to the unweighed case with the adding of weights on edges, which is presented in 3(a). The objective is to minimize the total weight on the set of disjoint paths that cover all abstracted trip vertices, hence is equivalent to a minimum weight minimum path cover problem.

Definition 4. Given a directed graph $G = (V,E)$, the **minimum weight minimum path cover** is a minimum path cover P that minimize the sum of the weights of the edges of paths of P, that is $\sum_{P_i \in P, i=1,2\ldots,|P|} \sum_{e \in P_i} w(e)$.

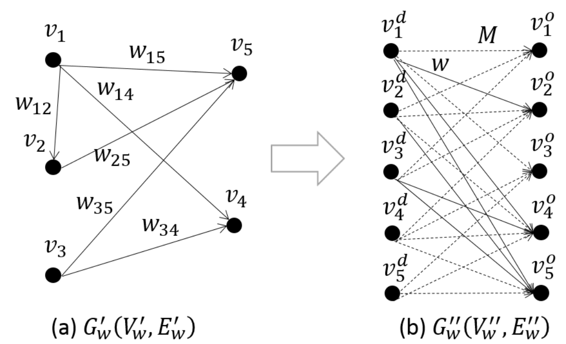


Figure 3: Illustration of Weighted Trip Integration

To solve this problem, simply transforming into a max-flow problem is not applicable. However, it is observed that the edges for matching on the bipartite graph G''_w also corresponds to the edges in the path cover on G'_w. As shown Proposition 1, finding a maximum matching will lead to a minimum path cover, hence a minimum weight maximum matching on the graph will correspond to the minimum weight minimum path cover problem [23]. Consequently, we can transform the graph $G'_w(V'_w, E'_w)$ into a complete bipartite graph $G''_w(V''_w, E''_w)$ illustrated in Figure 3(b), and instead solve a minimum weight maximum bipartite matching problem on G''_w. In G''_w, if the two trips are combinable, we use the same weight computed in Equation (4) on edges that connecting v_i^d and v_j^o. For all other edges, we assign the weight of a sufficient large number M. The Hungarian method can again be used to solve the problem. By removing the matching edges that contains the weight of M from the result, we arrive at the final solution that corresponds to the minimum weight minimum path cover problem.

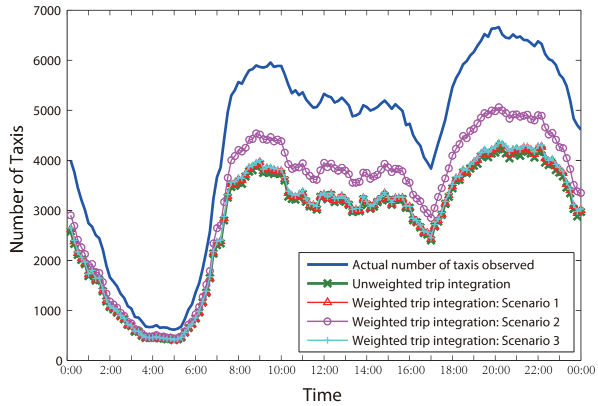
4. EXPERIMENT RESULTS

In this section, the experiment results of the theoretical optimal system performance are presented for both trip matching and trip integration. The results are carried out using the real world large-scale taxi data in New York City. The main idea of the experiment is to ensure all taxi trips are served at exactly the same time and location as in the data, and meanwhile, finding the optimal strategy that (1) minimizes the vacant trip cost (matching cost)for all taxis, and (2) finds the arrangement of minimum number of taxis with/without minimizing the vacant trip cost.

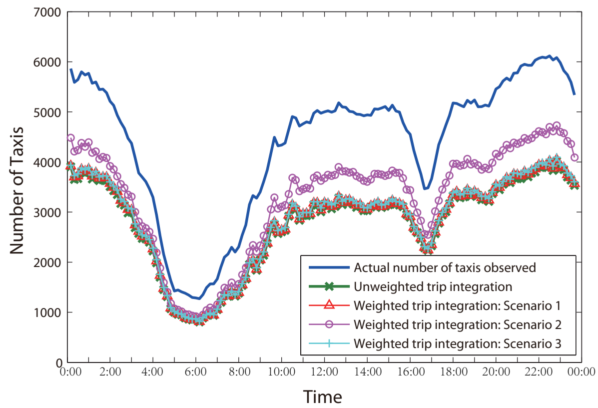
Since the taxi data from the actual taxi service system is the result of an inefficient driver-passenger matching strategy, it is not proper to directly implement the optimal matching on a large time interval for the data, although it is applicable to actual situations. This is because that the optimal matching is highly efficient in reducing empty trips, which leads to an increasing number of unserved taxis as time progresses. In real world cases, these unserved taxis could serve other potential trips, however, were not recorded in the data. Thus to evaluate the performance of the optimal matching, three short time period are selected (8:00-8:20, 12:00-12:20 and 18:00-18:20) which covers the morning and evening peaks and off-peak time The length of time interval ΔT is set to 5 minutes which results in 4 consecutive time intervals. Table 2 presents the optimal matching results for the three tested time periods for both weekday and weekend data.

From the optimal matching result, it is observed that the total taxis idle time (Scenario 1) can be reduced by as high as 85% $\sim$ 94%. For Scenario 3, in which the loss of revenue computation also involves the taxi idle time, the reduction is observed to be around 60% $\sim$ 75%. The result indicates that taxi drivers spent a significant amount of avoidable time in looking for passengers and a better scheduling for taxi-passenger matching can reduce taxi idle time to a great extent. Apparently, the optimal matching strategy can greatly enhance the level of taxi service by reducing both the waiting time for passengers and taxi drivers. On the other hand, when the objective is to minimize total vacant trip distance (Scenario 2), the improvement achieved in matching is less significant, which is about 15% $\sim$ 47%. The result is intuitive, as taxi drivers are more likely to roam around the latest drop-off location, while the optimal matching also tends to assign nearby trips to a taxi. Hence the reduction on the cost of extra travel distance may not be as prominent as the reduction on extra time cost spent on seeking a passenger. In general, we observe a huge gap between the amount of vacant trips traveled for the actual taxi service system and the theoretical optimal one. The gap is mainly due to the asymmetric information between taxi drivers and passengers.

For the experiments on trip integration, we consider the length of time interval $\Delta T = 10min$ and test both unweighted and weighted trip integration on the entire selected weekday and weekend data. Figure 4a,4b show the number of taxis required to serve all trips observed in data for both unweighted and weighted case in different scenarios. As expected, the unweighted trip integration yields the least amount of required taxis. The result shows that for most cases, only 2/3 of the observed taxis are actually needed to satisfy all the trips in the data. The whole system output (served trips) can be greatly boosted if all the taxis are fully utilized using an optimal trip integration strategy.



(a) Tested Weekday



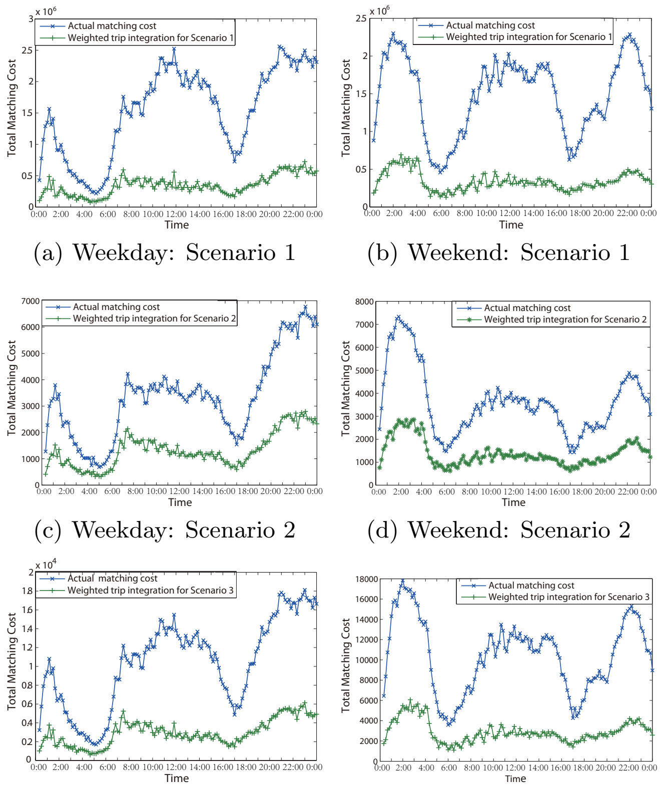
(b) Tested Weekend

Figure 4: Number of Taxis Required to Satisfy All Trips Observed in the Data

For weighted trip integration, the results show that the number of taxis required for Scenario 1 and 3 are only slightly higher than unweighted case. The optimal matching strategy is advantageous to provide the drivers with necessary information and achieves the greatest time saving during the period. While Scenario 2 requires more number of taxis, it still saves about 1/4 of total taxis. Figure 5a $\sim$ 5f present the comparison of matching costs for weighted trip integration of the entire day. It is found that for both weekday and weekend, 60% $\sim$ 80% of matching cost can be reduced for Scenario 1 and 3, and 45% $\sim$ 70% for Scenario 2. The reason that Scenario 2 needs more taxis and achieves lower cost reduction level is mainly because that the trip integration has more strict rules on using trip starting/ending times in verifying if trips can be combined, rather than distance. It should be noted that the reduction in Scenario 2 is larger than the optimal matching results even using fewer number of taxis. This is mainly because that (1) fewer taxis are considered, thus certain taxis that lead to higher vacant trip distance may be avoided; (2) a taxi is no longer restricted to be matched to only one trip, and the overall matching can be more efficient.

Table 2: Test Results for Optimal Matching

		Weekday (2009-10-07)			Weekend (2009-10-10)		
		8:00-8:20	12:00-12:20	18:00-18:20	8:00-8:20	12:00-12:20	18:00-18:20
Scenario 1	Original cost	3070962	3271368	2894850	2056667	3780612	2102510
	Optimal cost	427493	185171	446152	209284	335992	311095
	Reduction	86.08%	94.34%	84.59%	89.82%	91.11%	85.20%
Scenario 2	Original cost	7359.69	5304.16	6227.09	5889.28	7869.12	4699.95
	Optimal cost	6192.82	3679.93	4929.41	3126.54	5185.54	3358.95
	Reduction	15.85%	30.62%	20.84%	46.91%	34.10%	28.53%
Scenario 3	Original cost	21265.33	20193.33	19356.59	16041.03	25023.86	13482.54
	Optimal cost	8092.36	4564.26	6808.13	3986.64	6551.65	4770.45
	Reduction	61.95%	77.40%	64.83%	75.15%	73.82%	64.62%



(a) Weekday: Scenario 1
(b) Weekend: Scenario 1
(c) Weekday: Scenario 2
(d) Weekend: Scenario 2
(e) Weekday: Scenario 3
(f) Weekend: Scenario 3

Figure 5: Comparison of the Matching Costs for Weighted Trip Integration

Another interesting observation from the result is that the performance in terms of total matching cost of the actual system is significantly high from morning peak to afternoon (8:00 to 15:00), and late night (after 22:00). One reason is associated with the higher amount of operating taxis. Figure 6 shows the total number of operating taxis observed in the data. The increase in the number of taxis in the system is almost proportional to the increase in the actual matching cost Figure 5. This shows that certain level of competition exists in the taxi service system. The average matching cost remains almost the same level when both demand and supply are high. If compared with the results in the weighted trip integration, the optimal matching cost remains low and relatively stable during the entire day. This lead to an amazingly low average matching cost for each taxi when supply is abundant, which is very meaningful for the entire system.

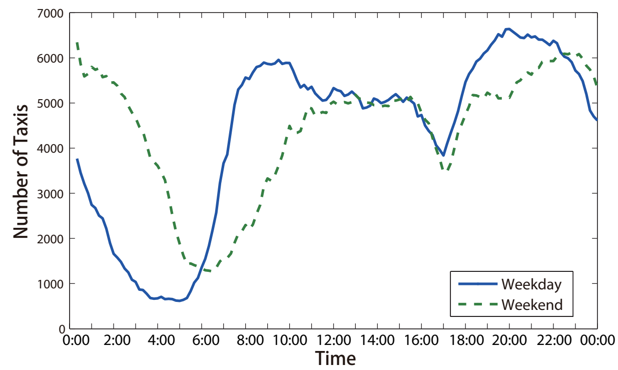


Figure 6: Total Number of Operating Taxis Observed in the Data

5. CONCLUSIONS

This paper presents the first study to quantify the efficiency level of the taxi service system in New York City using real world large-scale taxi trip dataset. A hypothetical system-wide recommendation mechanism is assumed that allow both taxi drivers and passengers to share their trip information. Two approaches, namely optimal matching and trip integration, are proposed to find the optimal strategy that minimizes the cost of vacant trips, and the number of taxis required to serve all observed trips. The optimization problems in the two approaches are transformed into equivalent graph problems and solved using polynomial time algorithms in graph theory.

The results show that the optimal matching can reduce 85% $\sim$ 94% total taxis idle time, 15% $\sim$ 47% total vacant trip distance, and 60% $\sim$ 75% total revenue loss of vacant trips when different objectives are considered. For trip integration, the results show that in most cases, only 2/3 of all taxis are sufficient to satisfy all the trips observed in the data. For weighted trip integration, the actual total matching cost in terms of idle time, vacant trip distance and revenue loss can be reduced by 45% $\sim$ 80% even when fewer taxis are used. Another observation from the results is that by regulating and coordinating the competition in taxi service, the system efficiency can be greatly improved.

The findings in this paper show that the actual taxi service system in New York City is far from an efficient one. The lack of sharing system-wide information between taxi drivers and passengers results in large amount of extra idle time and travel distance spent on vacant trips. Moreover, since it is shown that only 2/3 of the total taxis are sufficient to serve all observed trips, the critical taxi shortage issue is possible to be resolved by using a smarter arrangement.

Our analysis also suggests an urgent need to adopt system thinking into taxi recommendation or dispatching system design. The current decentralized taxi recommendation systems that involve a subset of all drivers and passenger trips might benefit specific taxi drivers or passengers, however, may not necessarily improve the entire system performance. Some current taxi hailing apps even make some taxi trips exclusive, which under certain situation will make the entire system worse. Thus new thinking from a system perspective should be considered towards building a more efficient and sustainable taxi service system.

Future works can be done to further extend the two approaches considered in this paper, and develop a taxi service recommendation and management systems for real world applications. By adopting a system consideration, such recommendation and management systems will lead to a more efficient taxi service system and a more sustainable urban environment.

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