Dynamic Simulation of Articulated Rigid Bodies with Contact and Collision

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presented by

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A couple of questions:
• Why did I choose this paper?
• What does “contact” vs “collision” mean?
• What are the theoretical bases of articulated rigid bodies?
• What is an “impulse”?
• What was “the previous work of Guendelman” about?
Outline

1. Theory behind:
   • Collisions and contacts
   • Time stepping
   • Articulated rigid body math

2. Core idea

3. Examples

4. Conclusions
Collisions and Contacts (1)

<table>
<thead>
<tr>
<th>Collisions</th>
<th>Contacts</th>
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<tbody>
<tr>
<td>• Bodies bounce off each other (elasticity factor)</td>
<td>• Bodies rest one stuck to the other</td>
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<td>• Motion of bodies changes discontinuously within a discrete time step</td>
<td>• Bodies slide (with or without friction) one upon the other</td>
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<td>• “Before” and “After” states need to be computed</td>
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Collisions and Contacts (2)

An example showing collisions and contacts:
Simulation loop

Traditional approach
• Update position and velocity
• Process collision
• Process contact

Guendelman approach
• Process collision
• Update velocity
• Process contact
• Update position
Traditional approach: problem

Example: block sliding down inclined plane

- Initially sliding down
- Update position and velocity $\rightarrow$ interpenetrating plane
- Process collision $\rightarrow$ velocity reflected
- No contact to process
- Next iteration $\rightarrow$ object bounces

SOLUTION: velocity threshold (Mirthic & Canny 1995)
Example: block sliding down inclined plane

- No collisions to process
- Update velocity → block gains downward velocity
- Process contact → stops normal motion
- Update position → slides down with no bounce
Time Stepping Theory (4)

An example showing the comparison:
Articulated Rigid Body Math (1)

Maximal coord.

\[(x_0, y_0, z_0, \theta_0, f_0, \psi_0) + (x_1, y_1, z_1, \theta_1, f_1, \psi_1) + (x_2, y_2, z_2, \theta_2, f_2, \psi_2) = \]

18 state variables

Generalized coord.

\[(x_0, y_0, z_0, \theta_0, f_0, \psi_0) + (\theta_1, f_1) + (\theta_2) = \]

9 state variables
Articulated Rigid Body Math (2)

The equations for rigid body evolution are:

1. \( \frac{d\vec{x}}{dt} = \vec{v} \)
   \( x = \) position, \( \vec{v} = \) velocity

2. \( \frac{d\vec{q}}{dt} = \frac{1}{2} \vec{\omega} \cdot \vec{q} \)
   \( \vec{q} = \) orientation, \( \vec{\omega} = \) angular velocity

3. \( \frac{d\vec{v}}{dt} = \vec{F} \)
   \( \vec{F} = \) net force, \( m = \) mass

4. \( \frac{d\vec{L}}{dt} = d(\vec{I} \cdot \vec{\omega}) = \vec{\tau} \)
   \( \vec{L} = \) angular momentum
   \( \vec{\tau} = \) net torque
Outline

1. Theory behind
2. Core idea:
   - Time integration and impulse theory
   - Prestabilization
   - Algorithm
3. Examples
4. Conclusions
These equations are time integrated according to:

\[
\begin{align*}
\vec{x}^{n+1} &= \vec{x}^n + \Delta t \cdot \vec{v}^n \\
\vec{q}^{n+1} &= \hat{\vec{q}}(\Delta t \cdot \omega^n) \vec{q}^n
\end{align*}
\]

Impulses are found and applied iteratively:
Goal: apply impulses to the rigid bodies BEFORE the integration step with the intention of achieving the target joint state after that integration step.

Position constraint equations:

\[ \vec{v}^{new} = \vec{v}^n \pm \frac{\vec{j}}{m} \]

\[ \omega^{new} = \omega^n \pm \frac{1}{I} (\vec{r} \times \vec{j}) \]
Algorithm

The algorithm is applied as follows:

• Process collisions (and velocity poststabilization)
• Integrate velocities (and velocity poststabilization)
• Resolve contacts and articulation prestabilization
• Update position (and velocity poststabilization)
Outline

1. Theory behind
2. Core idea
3. Examples
   • Black box definition of joints and constraints
   • Closed loops
   • Stacks of articulated rigid bodies
4. Conclusions
Joints - Basic

There are many kinds of basic joints:
Joints and Constraints

An example of joints combination:
Closed Loops (1)

An evidence of efficiency of closed loops:
Closed Loops (2)

Another evidence of efficiency of closed loops:
Closed Loops (3)

Another evidence of efficiency of closed loops:
Closed Loops (4)

Another evidence of efficiency of closed loops:
Closed Loops (5)

Another evidence of efficiency of closed loops:
Stack of Articulated Rigid Bodies (1)

First simulation involving large stacks:
Stack of Articulated Rigid Bodies (2)

Second simulation involving large stacks:
Outline

1. Theory behind
2. Core idea
3. Examples
4. Conclusions
Conclusions

- Any black box method for joint constraints
- Linearity both in the number of bodies AND in the number of constraints
- No special treatments of closed loops
- Advantage of pre vs post stabilization
Thanks for your attention!

References:

http://graphics.stanford.edu/~rachellw/
http://www.graphics.stanford.edu/~fedkiw/
http://graphics.stanford.edu/~jteran/

Questions?

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